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Inverse Dynamic Design Technique for Model Correction and Optimization

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The inverse dynamic design technique presented herein tunes a structural model to achieve a prespecified frequency response at certain critical points of the structure. The technique can correct structural models based on vibration response test data and also can optimize structural design. The technique works by partitioning the second-order matrix equations of motion into assigned and auxiliary degrees of freedom and then performs a parameter optimization to assign new frequency response data to the model. Design variable linking, symbolic or sparse matrix solutions, and closed-form gradients are used to streamline the computations. The technique still requires a large amount of computation, and thus is most applicable to small or moderate size dynamic models, such as coarse mesh prototype models, or models with repeated geometry or symmetry. Two small-order example problems illustrate the technique. The first example completely corrects the stiffness matrix of a corrupted analytical model using simulated measured (exact) frequency response data. The second example flattens the first resonance peak of a truss structure by assigning a specified partial frequency response to the model.

Nomenclature

E_r = error vector for frequency point r
H = system matrix
H^{-1} = inverse of system matrix with partitions (1, 1), (1, 2), (2, 1), (2, 2)
i = $\sqrt{-1}$
J = objective function
M, D, K = mass, damping and stiffness matrices with partitions (1, 1), (1, 2), (2, 1), (2, 2)
nf = number of frequency points in solution
p_0 = load vector with partitions 1, 2
q = displacement vector with partitions 1, 2
Q = weighting matrix
Re = real part of expression
t = time
ξ = design variables
ω = frequency, rad/s

Superscripts

T = matrix transpose
$*$ = complex conjugate

Introduction

UPDATING or adjusting finite element models, either for design purposes or to match experimental data, is a very important problem in design verification and optimization. Existing techniques for model updating and design optimization either assign eigenstructure[1–7] or optimize structural frequencies,[8–11] which are indirect approaches to achieving desired dynamic characteristics for the system. In eigenstructure assignment, it is usually not possible to accurately measure or define full mode shapes for a structure, and the problem of reducing the model size to fit the modal data is nonunique. Design methods that adjust natural frequencies are also indirect as they do not consider the overall response characteristics of the system. These methods may produce suboptimal design solutions. As an example of a model updating procedure, in Ref. 1 a corrupted dynamic model is adjusted using modal data and the success of the updating scheme is verified by comparison of the actual vs exact frequency response of the system. The basis of the inverse dynamic design (IDD) technique is that the frequency response information is indeed often the most useful definition of the dynamic characteristics of the model but that a direct method to modify the frequency response characteristics of dynamic systems is needed. Thus, rather than assigning a partial eigenstructure, the IDD technique directly assigns frequency response curves, through a bandwidth of interest, to certain critical points on the structure. The frequency response data completely defines the dynamic characteristics of critical points of the structure, that is, natural frequencies, damping, and modal participation.

The formulation of the IDD technique is based on a nonlinear parameter optimization to adjust the entries of the system matrices to minimize the difference between the existing vs the desired frequency response of the structure. This approach can take advantage of repetition in the entries of the stiffness matrix by "linking design variables"[12,13] to greatly reduce the number of optimization variables. To reduce computations, the system dynamic equation is partitioned into active and ancillary degrees of freedom and solved with smaller order symbolic or sparse matrix inversion. The gradient vector is obtained in closed form to further improve the speed and accuracy of the optimization.

The reason for selecting the design variables as direct entries of the updating matrices is to take advantage of the sparsity of structural matrices and, most importantly, to retain matrix connectivity and symmetry of the model; this is discussed in Ref. 1. Preserving the physical significance of the model is very important for design applications. Most model identification and updating techniques cannot retain the physical significance of the model or have restrictions such as only being able to update the stiffness matrix or computational limitations. The IDD technique will handle any system definition, uses the more easily definable or measured frequency response information, and, by linking design variables, will handle larger sized models. Geometric characteristics of the model can also be preserved.

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Other potential uses of the IDD technique (not done here) are designing active control systems and for structural damage detection. For active control, the controller could be designed using IDD by adding feedback matrices to the system equation to maintain positive definite collocated feedback for guaranteed stability. For damage detection, the structural damage would change the frequency response of the system, which could then be assigned to the undamaged model using IDD. The resulting changes in the stiffness matrix would identify the location of the damage, possibly more efficiently than manual inspection.

This paper is organized by first deriving the IDD technique, then giving recommendations on how to apply the technique, and then presenting illustrative examples on model updating and design optimization. A conclusion summarizes the features and limitations of the technique and suggests improvements.

Derivation of the Inverse Dynamic Design Technique

The IDD technique assigns a desired frequency response to critical degrees of freedom (DOF) of a dynamic or finite element model and is used when frequency response data or information is the most convenient approach to tune a structural model. A sinusoidal forcing function is assumed and only the prespecified or measured frequency response function at critical points and through a critical bandwidth is required.

There are two possible levels of parameter optimization that can be used with the technique. The first level, presented here, updates entries of the structural matrices. The second level optimizes primary design parameters (to be presented in a future paper), such as material or section properties, and can also perform a feature based parametric optimization. The complexity of the model and computational considerations drive the decision of the level of parametrization. Derivation of the basic equations is presented next.

Equation of Motion

The equations of motion that describe an n dimensional second-order linear dynamic system subject to a harmonic input are

$$M\ddot{q} + D\dot{q} + Kq = Re[p_0 \exp(i\omega t)] \tag{1}$$

where M, D, and K are the system matrices. These matrices are assumed to be symmetric, with M and D positive semidefinite and K positive definite. Often M is diagonal or equivalently, can be made diagonal. The forcing vector p_0 is constant, possibly complex, and defines the loading points on the structure. The D matrix has different forms depending on the type of damping modeled. In general, any damping configuration can be handled by the technique, including discrete dampers on the structure. As an example of modeling damping, D would be proportional to K to model a uniform material and structure. Modal damping would require extra computations to be used with the IDD technique and produces a fully populated D matrix that is not directly related to the geometry or material of the structure. Continuing, let

$$q(t) = Re[q_0 \exp(i\omega t)] \tag{2}$$

where q_0 is a constant complex vector. Substituting Eq. (2) into Eq. (1) gives

$$Re\Big\{\big[(K - \omega^2 M + i\omega D)q_0 - p_0\big]\exp(i\omega t)\Big\} = 0 \tag{3}$$

A solution to Eq. (3) is

$$(K - \omega^2 M + i\omega D)q_0 = p_0 \tag{4}$$

Now partition q_0 as $q_0 = [q_1\ q_2]^T$ where the subscript 1 denotes coordinates where the frequency response is to be assigned, and the subscript 2 denotes coordinates where the frequency response is not critical and, hence, is not assigned. Define

$$H^{-1} = (K - \omega^2 M + i\omega D)$$

which in partitioned form is

$$H^{-1} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} (K_{11} - \omega^2 M_{11} + i\omega D_{11}) & (K_{12} - \omega^2 M_{12} + i\omega D_{12}) \\ (K_{21} - \omega^2 M_{21} + i\omega D_{21}) & (K_{22} - \omega^2 M_{22} + i\omega D_{22}) \end{bmatrix} \tag{5}$$

Rewriting Eq. (4) gives

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{6}$$

Eliminating the q_2 coordinates from Eq. (6) gives

$$\big(H_{11} - H_{12}H_{22}^{-1}H_{21}\big)q_1 - \big(p_1 - H_{12}H_{22}^{-1}p_2\big) = \mathbf{0} \tag{7}$$

Inverting H_{22} as a function of frequency for each iteration of an optimization could be computationally very expensive for large-order systems. This constraint can be lessened as follows. First, impose a mild restriction on the D matrix that it have the same connectivity and symmetry as the K matrix, although it is not necessarily proportional to K. This restriction also makes sense on a physical basis because material-dependent damping will have the same connectivity as the stiffness matrix. The following approach is still possible without this constraint, although the size of problem that can be solved is reduced. From Eq. (5)

$$H_{22}(\omega) = \big(K_{22} - \omega^2 M_{22} + i\omega D_{22}\big) \tag{8}$$

Equation (8) shows that for a typical finite element structural model the H_{22} matrix is sparse, is symmetric, and has the same banding as the K matrix. Thus, its inverse can be found either using a sparse matrix solver, such as in Matlab, or for small problems, symbolically based. If the symbolic solution is obtained, the calculation of the H_{22}^{-1} matrix as a function of frequency will be very fast. Also, this symbolic solution holds throughout the optimization, even when the K, D, and M matrices change. Design variable linking will reduce the number of symbolic variables in the solution, and still further reduction in the number of symbolic variables, as well as design variables, can be obtained by assuming proportional damping. That is, $D = (c_1 K + c_2 M)$, where the c values are constants to be optimized. As an alternative to a one time full symbolic inversion, the symbolic inversion could be done each iteration of the optimization with the frequency as the only symbolic variable.

Computational requirements of the technique can also be reduced by carefully building the finite element model to minimize the bandwidth and number of nonzero entries in the stiffness matrix. This is done by minimizing the number of elements in the model and the number of contiguous elements per node and using repetitive geometry and symmetry and, if possible, using lower order elements (e.g., use plate or beam elements instead of solid elements). The next step in the procedure is to define the objective function, which is shown next.

Objective Function

An objective function that can be minimized to "assign" the $q_1(\omega)$ frequency response vector to the system can be written using Eq. (8) as

$$J = \sum_{r=1}^{nf} E_r^{*T} Q E_r \tag{9}$$

where Q is a diagonal weighting matrix, nf is the number of frequency points used to define the frequency response curves, and E_r is the error term, which is written as

$$E_r = \Big[\big(H_{11} - H_{12}H_{22}^{-1}H_{21}\big)q_1 - \big(p_1 - H_{12}H_{22}^{-1}p_2\big)\Big]_{\omega=\omega_r} \tag{10}$$

This form of error function was found to be much simpler to optimize than an error function defined as the least squares difference between the actual minus assigned frequency points. Note that if H_{22}^{-1} is found symbolically, only matrix multiplication is necessary to evaluate the error term, and no inversions are necessary. In many cases the p_2 vector will contain all or nearly all zeros further simplifying the computation of Eq. (10).

The design variables in the problem are some or all of the nonzero unique entries of the upper triangular parts of the system matrices M, D, and K and are automatically selected by specifying the coordinate DOF for which the system matrices are to be adjusted. Once a design vector is set up, the design variables will then be automatically linked by a searching algorithm to maintain identical properties for "families of elements" and to retain symmetry, connectivity, and geometric similarity in the updated model. A standard optimization routine, such as contained within the Matlab optimization toolbox, can be used to minimize Eq. (9). The only constraints on the design variables will be bounds on their magnitudes to ensure that sign changes or unreasonable magnitudes of matrix elements do not occur. The design variable bounds are given as follows:

$$\xi_j^L \leq \xi_j \leq \xi_j^U \qquad j = 1, 2, \ldots, ndv \tag{11}$$

where L and U represent the lower and upper bounds and ndv is the total number of design variables in the problem. No functional constraints are needed with this formulation.

A final requirement to make the optimization computationally feasible is to derive a closed-form gradient of the objective function. As shown next, the gradient can be obtained exactly without any additional function evaluations or matrix inversions. Since the second-order form of the system equations is used, matrix sparsity is preserved, and the Matlab sparse matrix functions take advantage of this to further streamline the computational process.

Gradient of the Objective Function

The closed-form gradient of the objective function (9) is necessary to make the technique computationally feasible for larger size models and is

$$\frac{\mathrm{d}J}{\mathrm{d}\xi_j} = \sum_{r=1}^{nf} 2Re(s_r) \tag{12}$$

where

$$s_r = E_r^{*T} Q \frac{\mathrm{d}E_r}{\mathrm{d}\xi_j} \qquad \text{and} \qquad \frac{\mathrm{d}E_r^{*T}}{\mathrm{d}\xi_j} = \left(\frac{\mathrm{d}E_r}{\mathrm{d}\xi_j}\right)^{*T}$$

The term $\mathrm{d}E_r/\mathrm{d}\xi_j$ is shown next and only involves matrix addition and multiplication to compute:

$$\frac{\partial E_r}{\partial \xi_j} = \left[\left(\left(\frac{\partial H_{11}}{\partial \xi_j} - \frac{\partial H_{12}}{\partial \xi_j} H_{22}^{-1} H_{21} - H_{12}\left(\frac{\partial H_{22}^{-1}}{\partial \xi_j} H_{21} + H_{22}^{-1}\frac{\partial H_{21}}{\partial \xi_j}\right)\right) q_1 + \left(\frac{\partial H_{12}}{\partial \xi_j} H_{22}^{-1} + H_{12}\frac{\partial H_{22}^{-1}}{\partial \xi_j}\right) P_2\right]_{\omega=\omega_r} \tag{13}$$

where

$$\frac{\partial H_{\alpha\beta}}{\partial \xi_j} = \frac{\partial (K_{\alpha\beta} - \omega^2 M_{\alpha\beta} + i\omega D_{\alpha\beta})}{\partial \xi_j}, \qquad \alpha, \beta = 1, 2$$

$$\frac{\partial H_{22}^{-1}}{\partial \xi_j} = -H_{22}^{-1}\frac{\partial H_{22}}{\partial \xi_j} H_{22}^{-1}$$

The gradients of the system matrices M, D, and K, with respect to the design variables, are simply matrices with all zeros, except with ones in the locations corresponding to the locations of the particular design variable. In programming the technique, the gradient matrices can be generated using the sparse matrix function in Matlab. Since both the objective function and gradient calculations require looping over the nf assigned frequencies, they can be computed together at each frequency point to save computer time and storage. The gradient calculation also loops on the design variables at each frequency point.

Comments on Using the Inverse Dynamic Design Technique

The computer algorithms to implement the method were developed within the framework of the Matlab software system. The codes were run on a Gateway 2000 P5-90 personal computer. The optimization step is performed using the Constr subroutine contained in the Matlab optimization toolbox. The Constr algorithm is a nonlinear optimization routine that finds the constrained minimum of a scalar multivariable function. The optimization is performed by arranging the design variables in a design vector containing the structural variables, such as mass, damping, and stiffness values (structural design variables are "linked" prior to this, and a reduced design vector is actually used for the minimization). The only constraints used in this technique are upper and lower bounds on each design variable; no functional constraints are needed. The algorithm that Constr uses to minimize the objective function is described in Refs. 14 and 15.

When selecting the DOFs to assign frequency response data, it is helpful to look for the columns of the stiffness matrix with the least number of zeros, that is, the greatest connectivity. Assigning the DOFs with the greatest connectivity will provide the greatest improvement in the model and will also place the maximum number of zero entries in the H_{22} matrix, which will speed up computations. Design performance requirements also dictate selection of the DOFs to be measured.

Example Problems

Two example problems are presented to show the features of the technique. The first example shows how the technique can be used for model updating. A 6-DOF structural model is corrected based on simulated test data and the IDD technique is compared to the constrained eigenstructure assignment technique of Ref. 1. The second example demonstrates how the IDD method can be used for structural design. A desired frequency response curve is assigned to a 16-DOF truss structure to minimize the vibration amplitude of the structure. Design variable linking is used to reduce computations.

Example Problem 1: Model Updating Using Frequency Response Data

This example resolves the model correction example of Ref. 1. In Ref. 1 an eigenstructure assignment method is used to assign measured (exact) vibration modes and frequencies to correct a 6-DOF spring-mass dynamic model. Here, however, we directly assign measured (i.e., simulated exact data without noise) frequency response data to correct the model. Specifically, we assign first one, two, and then as many frequency points as necessary to one and then two or more DOFs until the stiffness matrix is exactly corrected. Proportional damping ($D = 0.0002K$) is assumed for convenience. A sinusoidal force $f(t) = A\sin(\omega t)$ acts at DOF 4. The physical model is shown in Fig. 1.

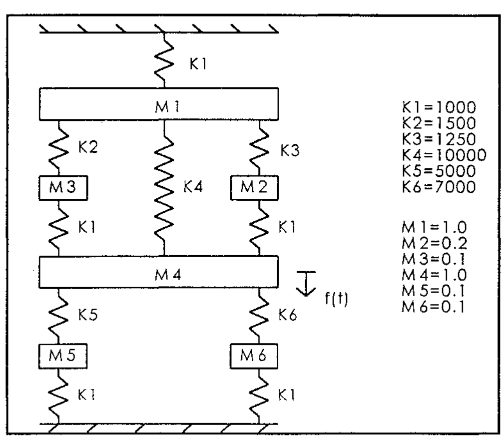

Fig. 1 Model with 6 DOF.

The corrupted and exact stiffness matrices are given as

$$K_{\text{corr}} = \begin{bmatrix} 15750 & -1300 & -1300 & -12000 & 0 & 0 \\ & 2150 & 0 & -850 & 0 & 0 \\ & & 2150 & -850 & 0 & 0 \\ & & & 22900 & -4200 & -5000 \\ & & \text{Symm} & & 5100 & 0 \\ & & & & & 5900 \end{bmatrix}$$

$$K_{\text{exact}} = \begin{bmatrix} 13750 & -1250 & -1500 & -10000 & 0 & 0 \\ & 2250 & 0 & -1000 & 0 & 0 \\ & & 2500 & -1000 & 0 & 0 \\ & \text{Symm} & & 24000 & -5000 & -7000 \\ & & & 0 & 6000 & 0 \\ & & & & 0 & 8000 \end{bmatrix}$$

The first step to correct the model was to assign the exact frequency response (FR) at DOF 4. A brief explanation of some details of using the technique is given as follows. Rows and columns 1 and 4 are switched in the system $\boldsymbol{M}$ and $\boldsymbol{K}$ matrices to put DOF 4 in the $\boldsymbol{H}_{11}$ position. This puts all the zero entries in the $\boldsymbol{H}$ matrix into the $\boldsymbol{H}_{22}$ partition. This simplifies the symbolic inverse of $\boldsymbol{H}_{22}$, which was obtained using Mathematica and is shown as

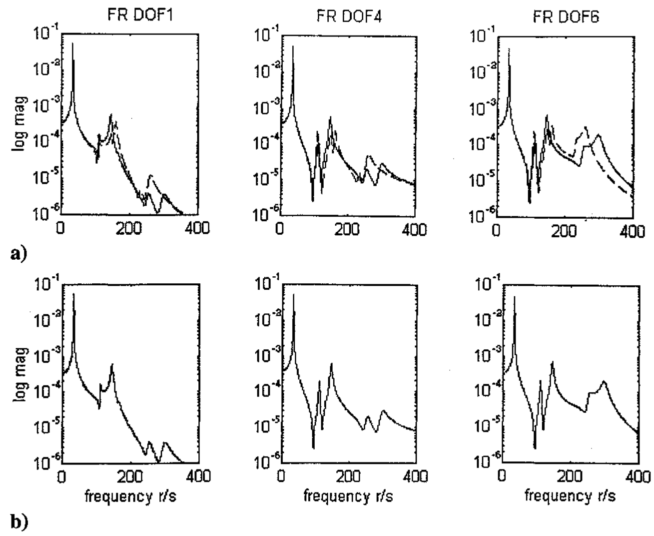

Fig. 2 Frequency response technique to correct 6-DOF model: a) exact/corrupted model and b) exact/updated model, with solid line exact model, dashed line corrupted/updated model.

$$\boldsymbol{H}_{22}^{-1} = \frac{1}{\det(\boldsymbol{H}_{22})} *$$

$$\times \begin{bmatrix} \begin{pmatrix} -h23^{\wedge}2^{*}h44^{*}h55+ \\ h22^{*}h33^{*}h44^{*}h55 \end{pmatrix} & (h13^{*}h23^{*}h44^{*}h55) & (-h13^{*}h22^{*}h44^{*}h55) & 0 & 0 \\ & \begin{pmatrix} -h13^{\wedge}2^{*}h44^{*}h55+ \\ h11^{*}h33^{*}h44^{*}h55 \end{pmatrix} & (-h11^{*}h23^{*}h44^{*}h55) & 0 & 0 \\ & & (h11^{*}h22^{*}h44^{*}h55) & 0 & 0 \\ & \text{Symmetric} & & \begin{pmatrix} -h23^{\wedge}2^{*}h22^{*}h55- \\ h11^{*}h23^{\wedge}2^{*}h55+ \\ h11^{*}h22^{*}h33^{*}h55 \end{pmatrix} & 0 \\ & & & & \begin{pmatrix} -h13^{\wedge}2^{*}h22^{*}h44- \\ h11^{*}h23^{\wedge}2^{*}h44+ \\ h11^{*}h22^{*}h33^{*}h44 \end{pmatrix} \end{bmatrix}$$

where the hij are the symbolic variables (i.e., the nonzero unique entries of $\boldsymbol{H}_{22}$) and $\det(\boldsymbol{H}_{22}) = -h13^{\wedge}2^{*}h22^{*}h44^{*}h55 - h11^{*}h23^{\wedge}2^{*}h44^{*}h55 + h11^{*}h22^{*}h33^{*}h44^{*}h55$.

By running cases with from 1 to 20 points, 15 points were found to be sufficient to represent the FR at DOF 4. In this problem the modes were relatively well spaced, and the resonant and antiresonant points and one or two points in between these were picked to represented the FR. Adding more points increased the solution time with negligible improvement in accuracy. How to model the FR curves using the minimum number of points needs further investigation. With 15 points assigned, the optimizer drove the objective function to zero in 546 iterations and took 9 min wall time to run on the personal computer. The eigenvalues of the model were corrected exactly, and the error in the norm of the stiffness matrix was reduced from 15% in the corrupted model to 3.8% in the updated model. The FR of DOFs 1, 4, and 6 for the corrupted and updated model are shown in Fig. 2. Figure 2a shows the FR of the exact (solid line) vs the corrupted model (dashed line), and Fig. 2b shows the FR of the exact and updated models.

Summarizing the first attempt at correcting the model, assigning the FR at DOF 4 corrected all of the natural frequencies of the model and corrected the FR at DOF 4 but left small errors in the stiffness matrix and in the FR of the remaining DOFs. This result points out the nonuniqueness of the solution that can occur by not using enough FR information to correct the model. However, by adding more FR information, a unique solution can be achieved. A weakness of other techniques of model correction, such as natural frequency adjustment or partial mode assignment techniques, is that they are unable to achieve a unique solution.

The second attempt to correct the model is to assign FR curves at DOFs 1 and 4. This optimization took 7 min wall time to run and corrected the stiffness matrix exactly. Thus, the FR curves for all DOFs in the corrected and exact models match and are not shown. In comparison to the constrained eigenstructure assignment technique of Ref. 1, that assigned three vibration modes and frequencies to correct the stiffness matrix, the IDD technique corrected the model by assigning 15 FR points at two DOFs. The advantages of the IDD technique presented here are that the FR at two DOFs is easier to measure than three mode shapes, the IDD technique is very simple and does not require expensive modal analysis equipment and software, and the IDD method can be extended to nonlinear systems. The disadvantage of the both techniques is that the optimization step limits the size of model that can be corrected.

Example Problem 2: Dynamic Design of a Four-Bay Truss

This example assigns an arbitrary desired frequency response to DOF 14 (node 7, y axis) of the four-bay truss structure shown in Fig. 3. The design goal is to reduce the cantilever type bending of the truss. A sinusoidal force $f(t) = A\sin(\omega t)$ also acts at this DOF.

The system mass and stiffness matrices are

$$M = [5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.6 \quad 5.65.6 \quad 5.6 \quad 5.5 \quad 5.5 \quad 5.3 \quad 5.3]^{*}10^{-5}$$

$$K = \text{symmetric with upper diagonals 5–15all zeros and other diagonals given as}$$

$$\text{upperdiag4}: [-363 \quad 0 \quad -363 \quad 0 \quad -363 \quad 0 \quad -363 \quad 0 \quad -363 \quad 0 \quad -363 \quad 0]$$

$$\text{upperdiag3}: [0 \quad 0 \quad 130 \quad 0 \quad 0 \quad 0 \quad 130 \quad 0 \quad 0 \quad 0 \quad 130 \quad 0 \quad 0]$$

$$\text{upperdiag2}: [0 \quad -725 \quad -259 \quad -65 \quad 0 \quad -725 \quad -259 \quad -65 \quad 0 \quad -725 \quad -259 \quad -65 \quad 0 \quad -725]$$

$$\text{upperdiag1}: [0 \quad 0 \quad -130 \quad 130 \quad -130 \quad 0 \quad -130 \quad 130 \quad -130 \quad 0 \quad -130 \quad 130 \quad -130 \quad 0 \quad 0]$$

$$\text{maindiag}: [984 \quad 790 \quad 984 \quad 790 \quad 984 \quad 790 \quad 984 \quad 790 \quad 984 \quad 790 \quad 984 \quad 790 \quad 622 \quad 790 \quad 363 \quad 725]$$

The vector of displacements is

$$q(t) = [x1 \quad y1 \quad x2 \quad y2 \quad \cdots \quad x8 \quad y8]^T$$

The mass matrix contains about 90% parasitic mass at each node of the truss. Proportional damping ($D = 0.0002K$) is assumed to simplify the example.

Design variable linking is used to keep the same form of the stiffness matrix but reduce the number of optimization design variables. Of the 44 nonzero upper triangular entries in the stiffness matrix, which would be the number of unreduced stiffness design variables, only eight remain after linking repetitive entries. This produces a large computational savings in the solution. Further, note that if the truss were 100 bays, still only 8 design variables would be needed.

The simple linking strategy used in this example does not guarantee that the updated stiffness matrix can be physically realized using tube-type truss members. This technique, however, produces a matrix that identifies the structural stiffness values that the designer should try to achieve in order for the structure to have the prespecified dynamic characteristics. Furthermore, this technique could be used for the design of smart structures by subtracting the original from the new K matrix where the difference would become the position feedback gain matrix for active control. As an alternative formulation, shape and area optimization of the truss could be performed. This would guarantee a physically realizable configuration.

Continuing with the example, rows and columns 1 and 14 of the system matrices are switched and the repeated inversion of H_{22} throughout the optimization is performed using the sparse matrix functions in Matlab. The "raw" symbolic inverse in this case was very long and more research into simplification is necessary to make it feasible to use. The frequency response of the existing structure at nodes 3 and 7 (DOF 6/14) in the y axis is shown as the solid line in Fig. 4. Seven frequency points from 20 to 50 r/s with a constant value of $-1.7606e{-}1 + 1.0144e{-}3^{*}i$ are assigned to DOF 14 to flatten the sharp resonance peak shown at 37 r/s in Fig. 4. Stiffness matrix values were arbitrarily bounded so that they would not change more than 50%. The optimization solution approximated the flat FR in 20–50 r/s region and shifted the first vibration mode frequency to 80 r/s. The FR of the adjusted model is shown as the dashed line in Fig. 4.

The peak displacement amplitude at the first resonance has been reduced by a factor of 6 by this FR assignment technique. The adjusted stiffness matrix changed by 47.5% and the eigenvalues changed by 21.4%. The original vs adjusted design vectors are shown as

$$DV_{(\text{orig})} = [790 \quad 130 \quad 65 \quad 725 \quad 984 \quad 259 \quad 363 \quad 622]^T$$

$$DV_{(\text{adj})} = [1128.4 \quad 150.3 \quad 32.5 \quad 1087.5 \quad 1476.0 \quad 388.5 \quad 181.5 \quad 832.7]^T$$

This example demonstrates that the IDD technique is a precise method of tailoring the frequency response of a structure nonetheless retaining the desired physical significance and connectivity of the dynamic model. More elaborate applications of frequency response assignment can be made, including assigning the phase response relationship between structural components.

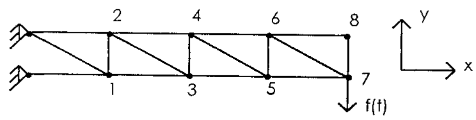

Fig. 3 Finite element model of a four-bay truss.

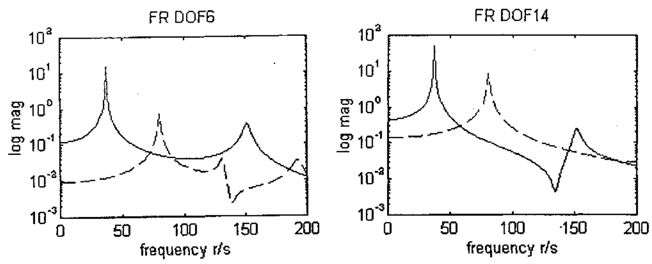

Fig. 4 Frequency response assignment for truss structure, with solid line original design and dashed line modified design.

Conclusion

The IDD technique is a departure from conventional structural optimization methods. It directly assigns frequency response data, which is often the easiest to obtain and most useful representation of the dynamic characteristics of a system. This approach does not require any eigenvalue or eigenvector or basis vector calculations or eigenvalue/vector derivatives, and does not use Guyan or mass reduction and, therefore, will be more accurate than existing techniques. Also, when the technique is used for model updating, expensive modal analysis software and elaborate test methods are not necessary.

The present limitation on the technique is that more research is needed to apply the technique to larger models. This could involve parallel computing. Simplification methods are needed to reduce the memory requirements when the symbolic inversion is used. The sparse matrix inversion makes the technique applicable to larger size models. The problem of noisy measurement data also needs to be investigated when the technique is used for model updating. Structural shape and properties optimization should be added to the technique for design purposes. Deriving feedback gain matrices for structural control applications using this technique is another area for research. A final suggestion is that if the heavy computations could be handled when design variables are not linked, the technique could be used for structural damage location.

Acknowledgments

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